Generation of tides

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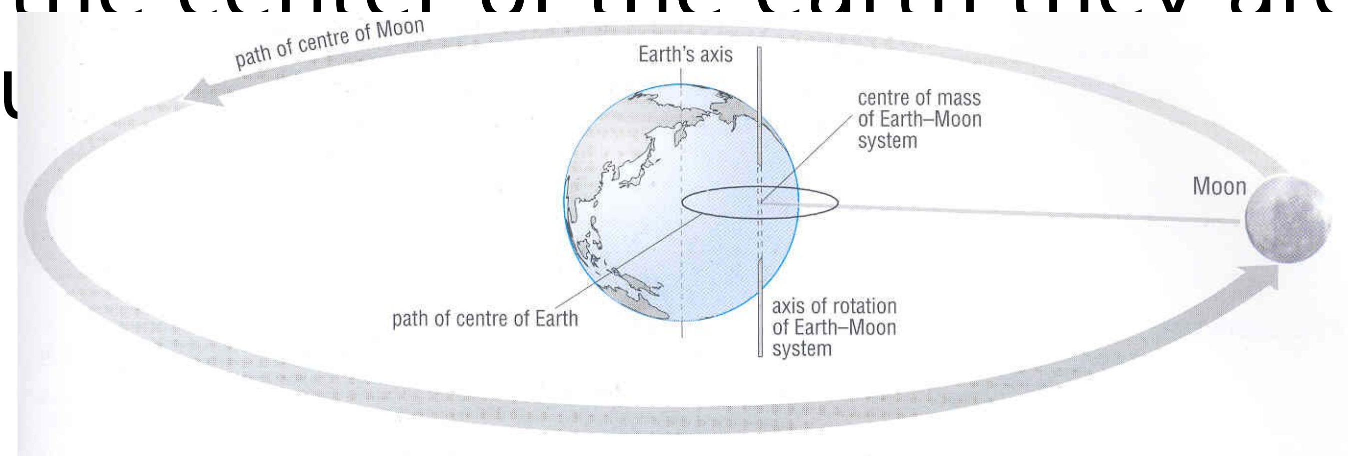
Generation of tides

- Tides are generated by a resultant force between the gravitational attraction of the moon and the centrifugal force due to the rotation of the earth.
- The tide-producing forces due to the sun is about 0.45 times that of the moon.
- Other planets are either too far away or small to generate any significant impacts.

Tide-generating forces

- For the earth-moon system as a whole is in equilibrium, the centrifugal force balances the gravitational force.
- At the center of the earth they are equ



Newton’s law of gravitation

F =Gm1m2/R2 m1

m2

R2

- Where m1 and m2 are the masses and R is the distance between the centres of masses m1 and m2

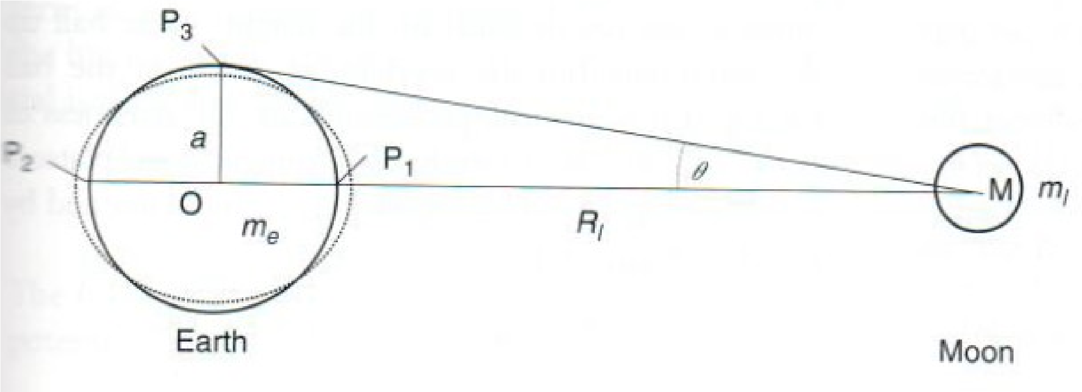


Figure 3.3 Diagram to show positions in the Earth–Moon system that are used to derive the tidal forces. The separation is distorted but the relative diameters of the Earth and Moon are to scale.

- In a simple case , assume the earth and the moon are in the same equatorial plane.
- Consider a particle of mass m at P1 on earth's surface. The attractive force by the moon is
- $Gmm_l/(R-a)^2$
- The force necessary for rotation can be equated with that of gravitational force at the centre of the earth O, which is Gmm_l/R^2

Source: Sea-Level Science (Pugh & Woodworth, 2014)

Resultant force

The difference between these is the tide-producing force at P_1:

$$G\frac{mm_1}{R_l^2}\left[\frac{1}{\left(1-\frac{a}{R_l}\right)^2}-1\right]$$

The term within the brackets can be expanded, making use of the approximations:

$$\left[\frac{a}{R_l}\right]^2 \ll 1 \quad \text{since } \frac{a}{R_l} \approx \frac{1}{60}$$

and expanding the expression $[1/(1-\alpha)^2]\approx(1+2\alpha)$ for small α to give a net force towards the Moon of:

$$\text{Tidal force at } P_1 = \frac{2Gmm_l a}{R_l^3} \qquad (3.2)$$

- Similarly, it can be shown that at a diametrically opposite point P2,

$$\text{Tidal force at } P_2 = -\frac{2Gmm_l a}{R_l^3}$$

$$\frac{G m m_l}{R_l^2}\left[\frac{1}{1+\frac{2a}{R_l}} - 1\right]$$

$$= \frac{G m m_l}{R_l^2}\left[\frac{1}{\frac{R_l+2a}{R_l}} - 1\right]$$

$$\left[\frac{R_l}{R_l+2a} - 1\right]$$

$$\left[\frac{R_l - (R_l+2a)}{R_l+2a}\right]$$

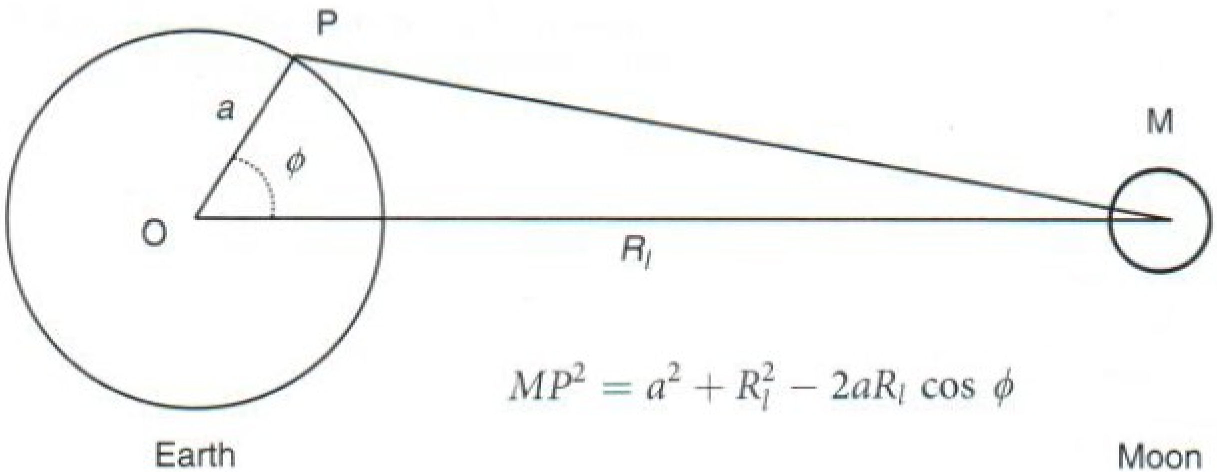
$$\left[\frac{2a}{R_l+2a}\right.$$

$$\left[\frac{2a}{R_l}\right.$$

$$= \frac{G m m_l}{R_l^2}\,\frac{2a}{R_l} = \frac{2 G m m_l a}{R_l^3}$$

Tidal potential

- The next aim is to derive an expression for ‘Equilibrium Tide’ .
- For a fuller development, it is convenient to use potential, instead of force. (The gradient of potential is the force). Potential can be expressed as
- $\Omega p = - G\ ml/distance$



$$MP^2 = a^2 + R_l^2 - 2aR_l \cos \phi$$

Terms: Force = 1. No force
2. Constant Force – orbital motion
3. Tidal Potential

we have:

$$\Omega_p = -\frac{Gm_l}{R_l}\left[1 - 2\frac{a}{R_l}\cos\phi + \frac{a^2}{R_l^2}\right]^{-\frac{1}{2}}$$

The term in brackets may be expanded as a series of Legendre polynomials:

$$\left[1 + \frac{a}{R_l}P_1(\cos\phi) + \frac{a^3}{R_l^3}P_2(\cos\phi) + \frac{a^3}{R_l^3}P_3(\cos\phi) + \cdots\right] \quad (3.7)$$

The terms $P_1(\cos\phi)$ etc. are Legendre polynomials:

$$P_1(\cos\phi) = \cos\phi$$
$$P_2(\cos\phi) = \frac{1}{2}(3\cos^2\phi - 1)$$
$$P_3(\cos\phi) = \frac{1}{2}(5\cos^3\phi - 3\cos\phi)$$

The tidal forces represented by the terms in this potential are calculated from their spatial gradients $-\nabla(P_n)$. The first term in Equation 3.7 is constant (except for variations in R_l) and so produces no force. The second term produces a uniform force parallel to OM because differentiating with respect to ($a \cos\phi$), the projected vector along OM, yields a gradient of potential:

$$-\frac{\partial\Omega p}{\partial(a\ \cos\phi)} = -\frac{Gm_l}{R_l^2}$$

This is the force necessary to produce the acceleration in the Earth's orbit towards the centre of mass of the Moon–Earth system. **The third term is the major tide-producing term**. For most purposes the fourth term may be neglected because $\frac{a}{R_l} \approx \frac{1}{60}$. So too may all the higher terms. The effective tide-generating potential is therefore written as:

$$\Omega_p = -\frac{1}{2}Gm_l\frac{a^2}{R_l^3}\left(3\cos^2\phi - 1\right) \quad (3.8)$$

The force on the unit mass at P corresponding to the potential may be resolved with two components:

- Tide-generative force can be split into horizontal and vertical components. Vertical component is small compared to gravity and the horizontal component, ‘tractive force’, wh

Vertically upwards : $$\frac{-\partial\Omega_p}{\partial a} = -2g\Lambda_l\left(\cos^2\phi - \frac{1}{3}\right) \tag{3.9}$$

Horizontally in the direction of increasing ϕ :

$$-\frac{\partial\Omega_p}{a\partial\phi} = -g\Lambda_l \sin 2\phi$$

where $\Lambda_l = \frac{3}{2}\frac{m_l}{m_e}\left(\frac{a}{R_l}\right)^3$

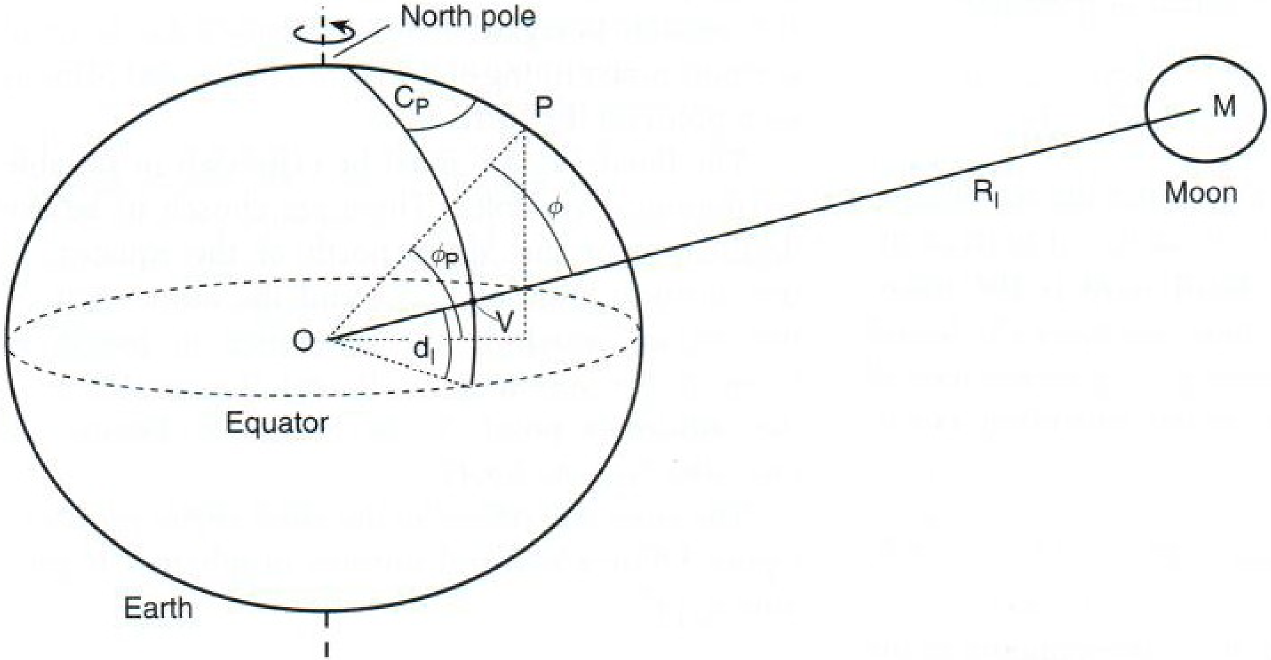


Figure 3.6 The three-dimensional location of a point P on the Earth's surface relative to the sub-lunar position. The angle POV is ϕ. T difficult diagram to convert into three dimensions, but it helps to recall that V is on the surface of the Earth and on the line O to M.

The angle ϕ is related to the other angles shown in Figure 3.6 by a standard formula in spherical trigonometry [4]:

$$\cos \phi = \sin \phi_p \sin d_l + \cos \phi_p \cos d_l \cos C_p$$

Hour **angle**, when expressed in hours and minutes, is the time elapsed since the celestial body's last transit of the observer's meridian. The **hour angle** can also be expressed in degrees, 15° of arc being equal to one **hour**. See also right ascension.

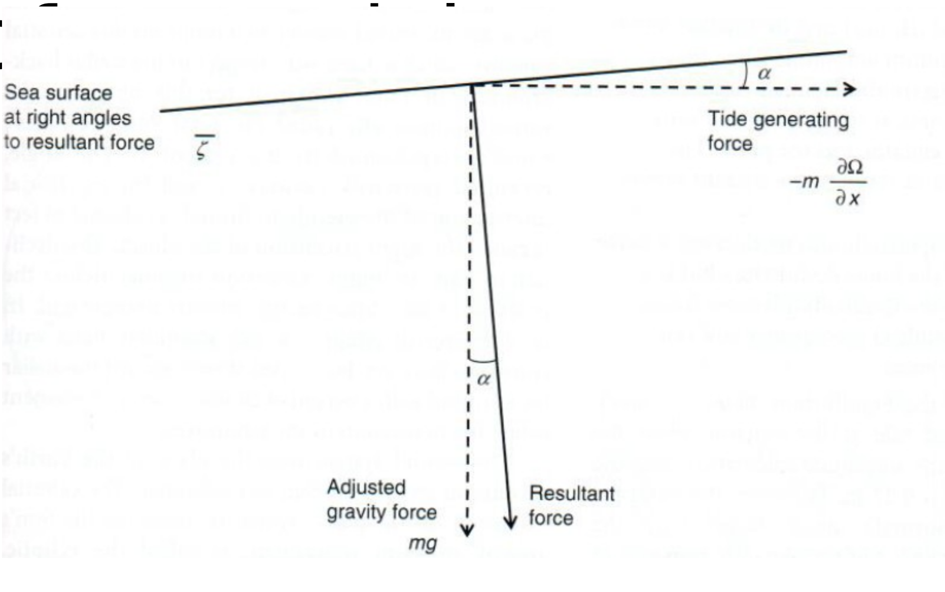
Substituting for $\cos\phi$ in Equation 3.8, with some further rearrangement, eventually yields:

$$-\Omega_p = \frac{3}{2} a g \frac{m_l}{m_e} \left(\frac{a}{R_l}\right)^3 \left[\begin{array}{l} \frac{3}{2}\left(\sin^2 d_l - \frac{1}{3}\right)\left(\sin^2\phi_p - \frac{1}{3}\right) \\ \quad + \frac{1}{2}\sin 2d_l \sin 2\phi_p \cos C_p \\ \quad + \frac{1}{2}\cos^2 d_l \cos^2\phi_p \cos 2C_p \end{array} \right] \tag{3.10}$$

As before, d_l is the lunar declination, ϕ_p is the latitude of P, and C_p is the hour angle of P.

Deriving an expression for sea surface elevation

- Because of the tide-generative force, the sea surface gets tilted by α.
- The force, g tan α , can be equated with the gradient o



$$\tan \alpha = -\left(\frac{\partial \Omega_p}{\partial x}\right) \Big/ g$$

and also:

$$\tan \alpha = \left(\frac{\partial \bar{\zeta}}{\partial x}\right)$$

so that:

$$g\left(\frac{\partial \bar{\zeta}}{\partial x}\right) + \left(\frac{\partial \Omega_p}{\partial x}\right) = 0 \text{ or } \frac{\partial}{\partial x}\ (g\bar{\zeta} + \Omega_p) = 0$$

and similarly:

$$\frac{\partial}{\partial x}(g\bar{\zeta} + \Omega_p) = 0$$

Integrating over a finite area gives:

$$g\bar{\zeta} + \Omega_p = \text{constant} \tag{3.11}$$

If the integral is taken over the whole area of the sphere's surface so that the total volume of water is conserved,

Equilibrium Tide

then the constant is zero. Applying this condition to Equation 3.10 the Equilibrium Tide becomes:

$$\bar{\zeta} = a\frac{m_l}{m_e}\left[C_0(t)\left(\frac{3}{2}\sin^2\phi_p - \frac{1}{2}\right) + C_1(t)\sin 2\phi_p + C_2(t)\cos^2\phi_p\right]$$

where the time dependent coefficients are:

$$\begin{aligned} C_0(t) &= \left(\frac{a}{R_l}\right)^3\left(\frac{3}{2}\sin^2 d_l - \frac{1}{2}\right) \\ C_1(t) &= \left(\frac{a}{R_l}\right)^3\left(\frac{3}{4}\sin 2d_l \cos C_p\right) \\ C_2(t) &= \left(\frac{a}{R_l}\right)^3\left(\frac{3}{4}\cos^2 d_l \cos 2C_p\right) \end{aligned} \tag{3.12}$$

These three coefficients characterise the three main species of tides:

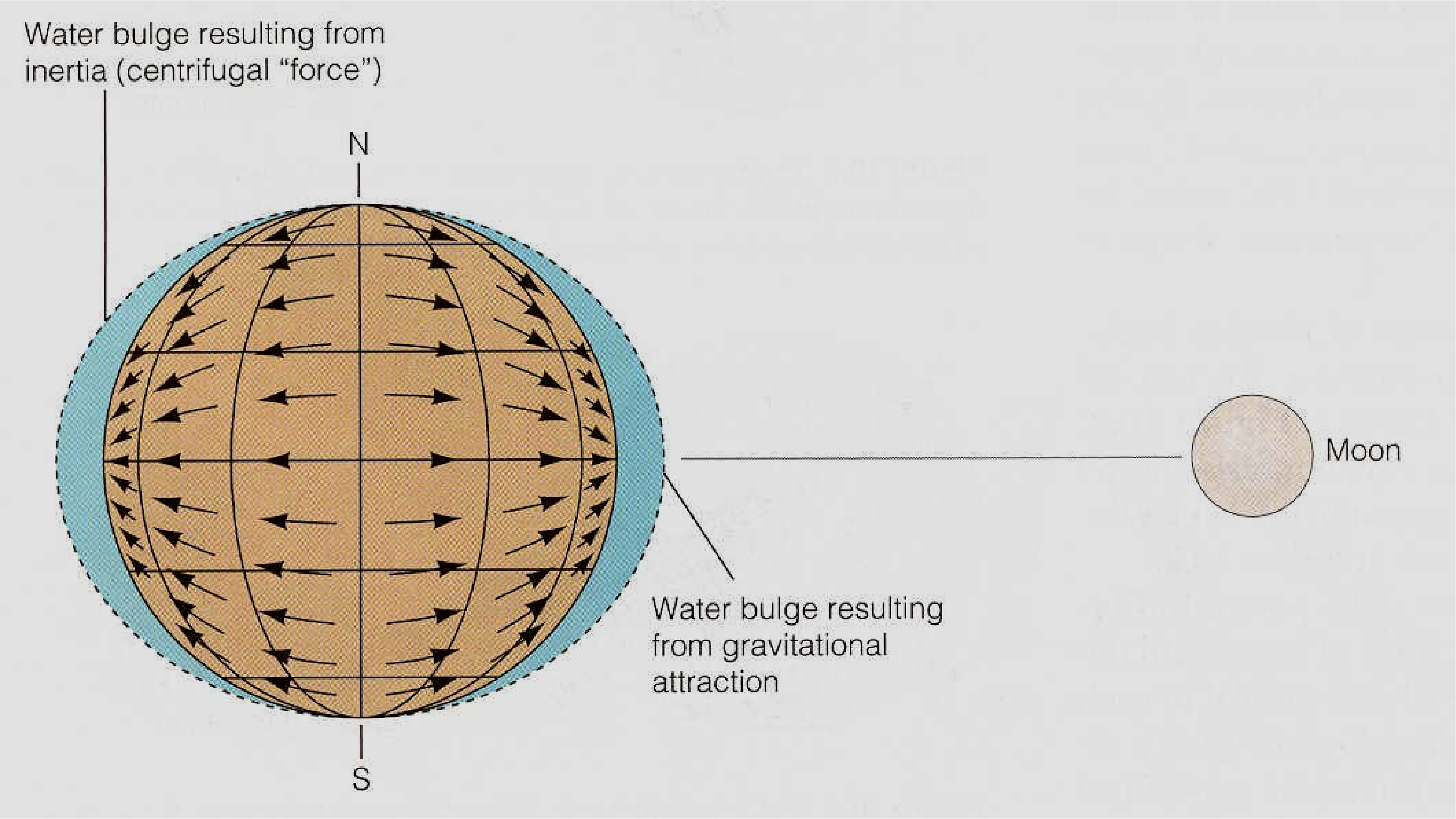
- the long period species,
- the diurnal species at a frequency of one cycle per day ($\cos C_p$) and
- the semidiurnal species at two cycles per day ($\cos 2C_p$).

The magnitudes of all three species are modulated by a common term, which varies inversely as the cube of the lunar distance R_l.

Equilibrium Tide due to the sun

- Equilibrium Tide due to the sun can be obtained in a similar way with m*l* , Rl and dl replaced by ms, Rs and ds . The amplitudes are smaller by a factor of 0.46

Force imbalance in earth-moon system creates tidal bulges



Historical perspectives of development of theories on tides

- Newton - Equilibrium theory
- Daniel Bernoulli, Euler, Laplace - Dynamic theory of tides
- Darwin - Tidal constituents (M2, S2 etc., the notation of subscripts)
- Doodson (1921) Large no. (more than 300) of tidal constituents, Tidal prediction, Doodson numbers, Doodson filter)

Cartwright (1971) Fourier analysis derived from emphemerides (solar and lunar positions)

Le Provost (1978, 1995) - Physical model of tides in the English channel (1978), global tidal model (1990 ‘s)

Equilibrium theory of tides

- Put forward by Newton at the end of seventeenth century. Equilibrium tide is the tide generated by tide-generating forces, which is the instantaneous response of the ocean (no inertia, no friction) in an earth covered fully with ocean.
- In practice, observed tides do not follow the theory of equilibrium tides. The presence of continents and shallow coastal regions alter the propagation of tides in the ocean

Tides in the Ocean and Equilibrium Tides

- Tides in the real ocean are much different than the equilibrium tides.
- In spite of the limitations in theory of Equilibrium Tides, it is found that frequencies present in equilibrium tide match those in real ocean
- However, some of the predictions using equilibrium theory are found to be consistent with observations
  (i) Spring tides will occur at new moon or full moon
  (ii) The range of spring tides will be at least three times that of the neap tide.

Dynamic theory of tides

- Bernoulli, Euler and Laplace
- Laplace's tidal equations (LTE)

- But, practical difficulties occur due to the geometry of the ocean basins, variations in depths etc. , which are overcome in the present-day numerical models

Tides in the ocean, solid earth and the atmosphere

- Tides in the ocean-
- Tides on the solid earth – The earth is not absolutely rigid, but slightly elastic. The earth undergoes slight bulges due to the attraction of the moon and the sun. Earth tides are measured using strain gauges kept beneath the earth.
- Atmospheric tides – Radiational more than gravitational. It can be found in atmospheric pressure (< 2 mb normally)